AFOSR-TR-97

0647

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE December 31, 1996	3. REPORT TYPE AND DATES COVERED Final Report 1/1/94 to 12/31/96

4. TITLE AND SUBTITLE

Nonlinear Wave Propagation

5. FUNDING NUMBERS

F49620-94-1-0120

AFOSR-TR-97

0647

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1534615

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10. SPONSORING MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

There were a number of significant accomplishments during the past 3 years. In this period, 12 papers were published in refereed journals, 4 papers were published as chapters in books and conference proceedings, 7 papers were accepted for publication, and 28 invited lectures were presented. This research has produced a number of fundamental contributions in: nonlinear fiber optics, specifically wave length division multiplexing, new classes of solutions to multidimensional nonlinear wave equations of physical significance, direct and inverse scattering, the propagation of magnetic spin waves in thin ferromagnetic films, and computational and effective chaos associated with a class of integrable nonlinear wave equations. A summary of this effort is contained in this report, as well as a list of all publications and invited lectures.

1/21/97

14. SUBJECT TERMS	15. NUMBER OF PAGES 17
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT

Nonlinear Wave Propagation
Air Force Office of Scientific Research
Grant F49620-94-1-0120

Final Report
1 January 1994 – 31 December 1996

December 31, 1996

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OBJECTIVES

To carry out fundamental research investigations involving the nonlinear wave propagation that arises in physically important systems, in particular: nonlinear optics, fluid dynamics, ferro-magnetic systems and related issues such as inverse scattering.

ACCOMPLISHMENTS/NEW FINDINGS

i) Nonlinear Optics:

One of the most exciting achievements in the field of nonlinear optics has been the development of optical soliton communication systems which are capable of high speed data transmission. In thin optical fibers the fiber can support localized pulse/soliton propagation in the anomalous-dispersion regime. It is well-known that the nonlinear Schrödinger equation (NLS) governs the propagation of such waves. The NLS equation supports multisoliton solutions. In order to deal with the realistic technological application, the NLS model is modified by incorporating damping and amplifiers.

Single channel systems supporting single soliton waves involve communication systems in which the solitons are widely separated and there is no possibility of mutual soliton

interactions. Laboratory demonstrations have already proved that such systems are capable of effectively transmitting data at relatively high rates. However, for future communication systems, researchers must develop systems capable of significantly higher data rates. Towards this end an effective multisoliton based system, capable of rapidly transmitting data in numerous wavelength channels, is needed. Such systems are usually referred to as wavelength division multiplexed (WDM) soliton systems.

Our research is aimed at the study of multisoliton interactions in nonlinear optics. With regard to soliton based communication systems, we have developed an effective theory describing soliton interactions in both physical and frequency space.

In our first papers we analyzed multisoliton interactions in an ideal fiber; i.e. soliton interactions in the NLS equation. In general, the formulae are complicated and unwieldy. However, in the limit of large frequency separations, which is the physically relevant limit, the analysis greatly simplifies. We found that solitons always remain widely separated in frequency space, even when they interact strongly in physical space. Numerically we have observed and confirmed this phenomena. The solitons do remain widely separated in Fourier space, even when the frequency separation is, numerically speaking, moderate. Consequently this limit is expected to be effective and useful in physical environments; in fact, this is the parameter regime which is currently being used in laboratory experiments. See also figures 1–2 where this situation is clearly depicted. Figure 1 represents a typical soliton interaction in physical space and figure 2 illustrates the interaction in frequency (i.e. Fourier transform) space at a typical value of the parameters relevant to the asymptotic theory. Note that even while the solitons interact strongly in physical space, the interaction only results in slight frequency modifications in Fourier space.

Subsequently we discovered that significant perturbations can be generated in a different frequency channel from the individual solitons In fig 2 note the small bumps in the adjacent frequency channels). We found that the perturbations in this new frequency channel are located in specific frequency regimes, and that these new perturbations are excited in precisely the frequency regimes associated with four wave mixing interactions (FWM) contributions. We have subsequently developed a comprehensive theory of FWM and multisoliton interactions. It should be noted that, in fact, such a situation is generic to any soliton system which has underlying carrier waves. Hence NLS type systems (and vector NLS systems) can exhibit such a phenomenon, but Korteweg-deVries type systems, which do not have carrier waves associated with them, will not.

In the ideal fiber we show that as solitons interact, the FWM contributions grow from a zero background and then decay back to zero. A typical situation is depicted in figs 3-5 [from our paper on FWM in ideal fibers—see paper #17 in the 'publications accepted in journals'

section of this report]. The figures show the solitons as they interact in physical space (fig 3) and the corresponding results in frequency space (fig 4). As in fig 2 above, note the small frequency contributions in fig 3 that grow and decay in the adjacent frequency channels to the main solitons. In the inset figure of fig 3 these small contributions are magnified and the numerical (solid line) and our analytical (dashed line) theory are compared—with excellent agreement demonstrated. We have also shown that whenever the FWM contributions are located on a main soliton frequency channel, there is a particularly significant effect on the perturbation to the soliton frequency. In an ideal fiber, i.e. governed by the pure NLS theory, the FWM contributions grow and then decay. However our results indicate that this will not be the situation in realistic systems.

We subsequently studied the NLS equation with damping and amplification present. We have developed a detailed analytical theory describing the effects of multisoliton collisions and, in particular, the effects of FWM. Our analytical results are confirmed by extensive numerical simulations by us as well as by well-known scientists in this field (J. Mollenauer and colleagues). The essential point is the following. In realistic systems, damping must be included. Amplifiers are placed at periodic intervals in order to compensate for the damping. It turns out that the amplifiers can and do resonate with the FWM contributions. The result is that FWM signals are magnified; they grow and then saturate to become a nontrivial state. In figure 6–7 (figure 6–7 are from our paper on FWM with damping and amplification present; see paper #12 in 'publications accepted in journals' section of this report), a typical situation is illustrated: fig 6 is the physical space description and fig 7 is the corresponding frequency space description. The difference from the ideal fiber case is striking. From a communications standpoint such FWM contributions are undesirable and one of our future research problems involves how to control/eliminate this phenomenon. It is also worth noting that such distinct FWM contributions might be a positive feature e.g. in a situation where one may wish to create a significant signal in an adjacent frequency channel.

This work is relevant for researchers interested in WDM fiber optic communications. As such, this has important applications to both defense and civilian technologies. It also has ramifications in fluid dynamics and indeed any area where the NLS equation is central in the description of the physical phenomena.

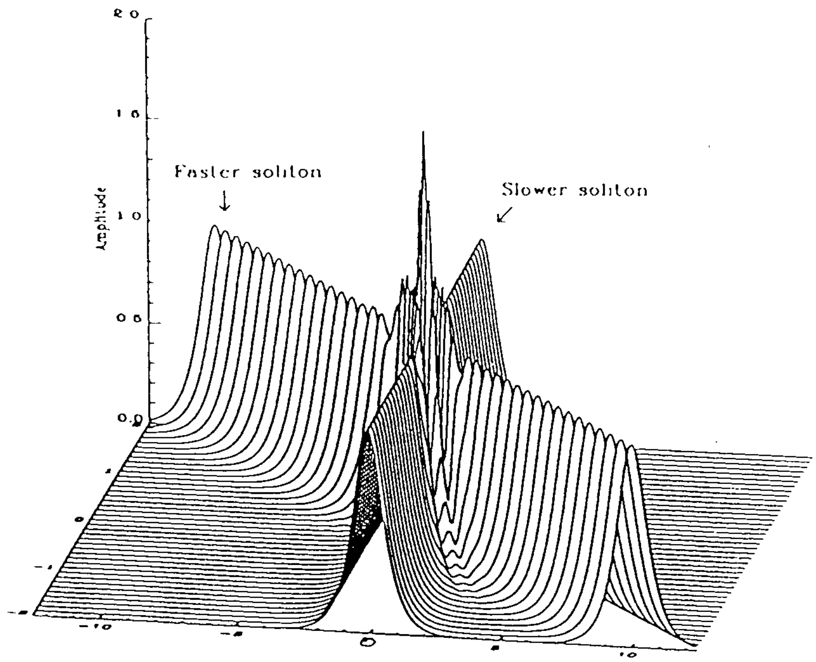

Figure 1

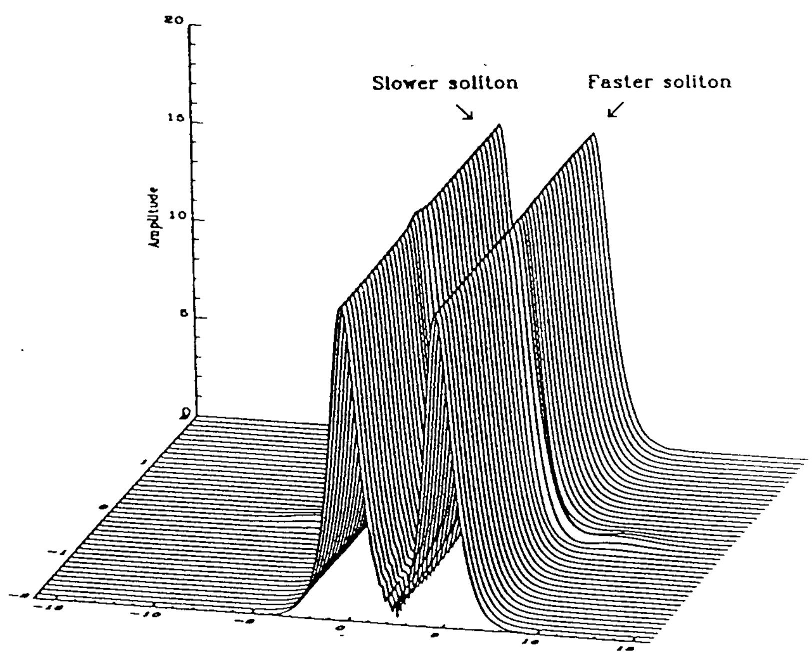

Figure 2

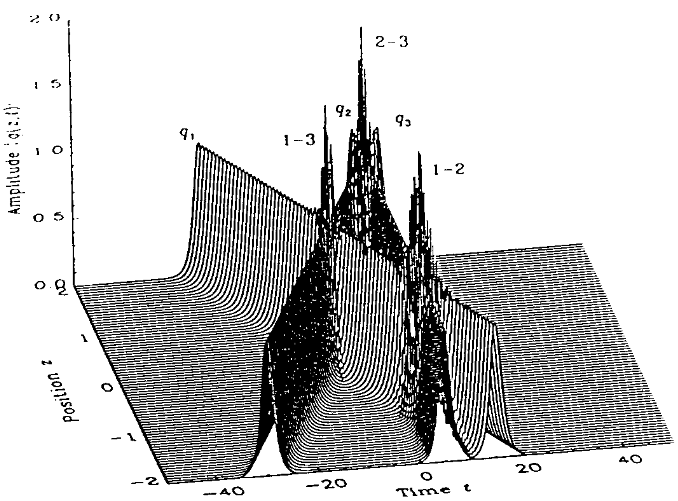

Figure 3

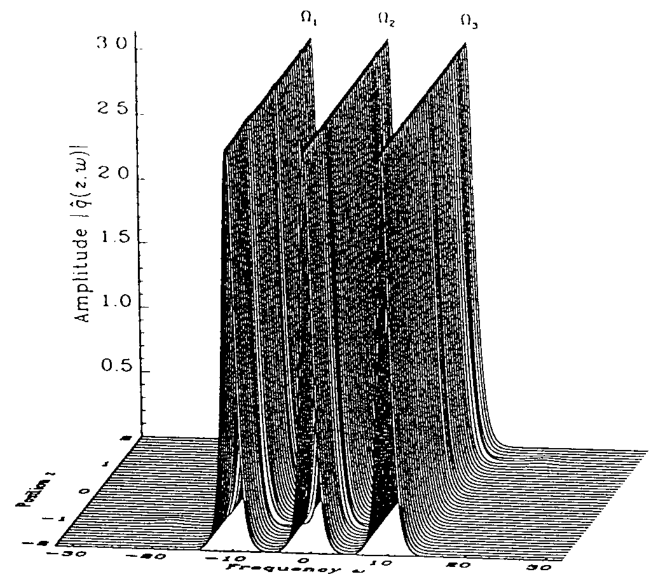

Figure 4

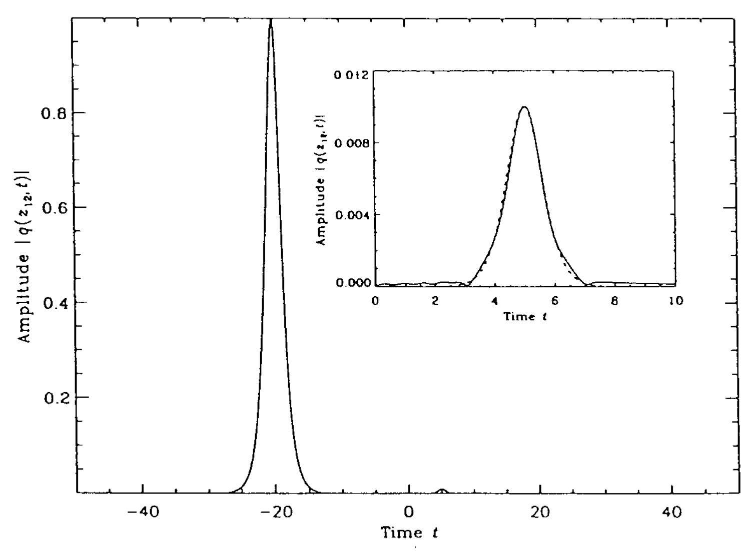

Figure 5

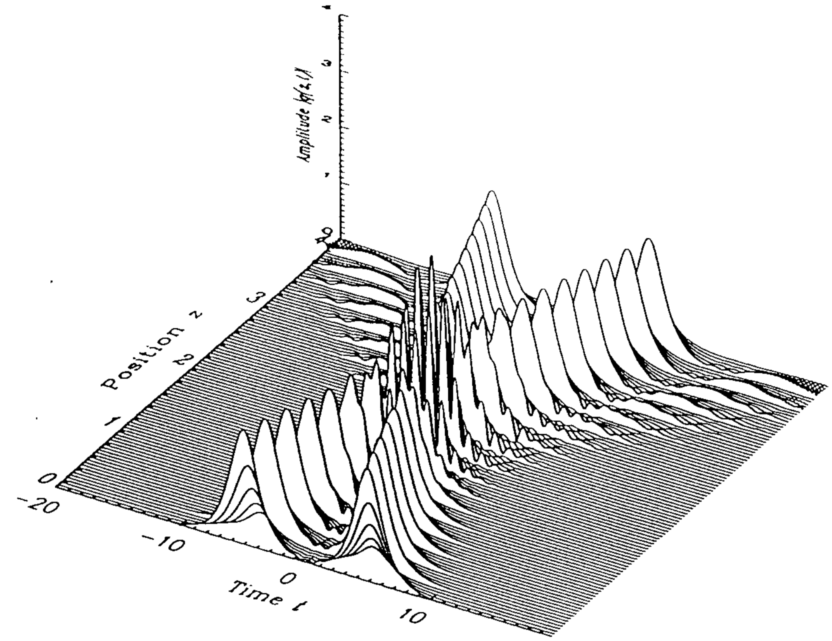

Figure 6

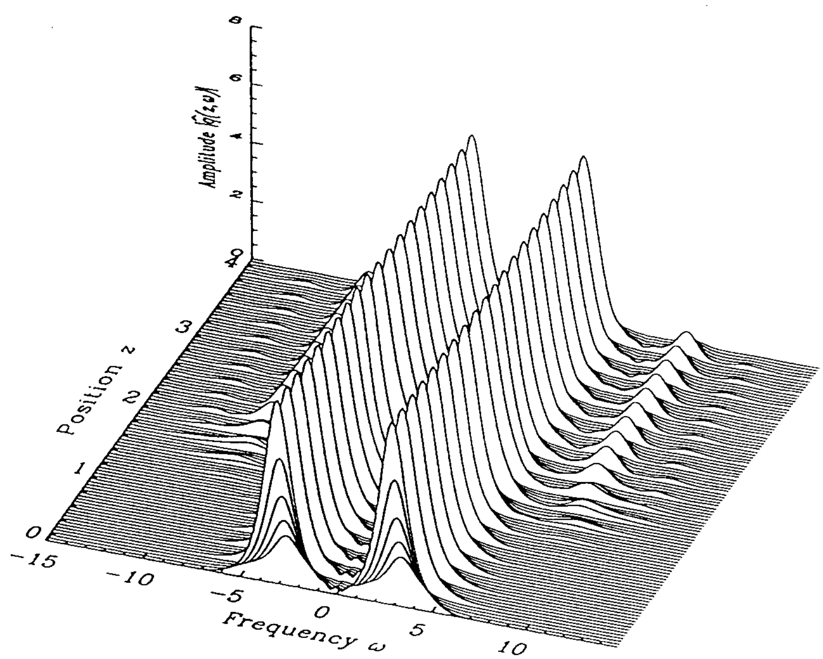

Figure 7

ii) Multidimensional Nonlinear Wave Equations and the Inverse Scattering Transform

The inverse scattering transform (IST) allows one to linearize and obtain global information about certain physically significant nonlinear wave equations. For example, in one dimension, IST applies to such important equations as the nonlinear Schrödinger equation (NLS) which is a centrally important equation in nonlinear optics and ferromagnetics; the Korteweg-deVries (KdV) equation which governs long waves in free surfaces (with or without surface tension) and internal stratified waves in fluids; and the sine-Gordon equation which models waves in Josephson junctions in superconductors etc.

Particularly significant is that the IST method can also be applied to certain multidimensional equations in two space-one time dimension (2+1), a number of which are physically important. In 2+1 dimensions, well-known physically important equations solvable by IST are the Kadomtsev-Petviashvili (KP) equation, the Davey-Stewartson (DS) system, and the three wave interaction equations in two dimensions.

In our earlier work we found solutions to a variety of initial value problems with rapid decay at infinity. An important special class of solutions which we obtained are "lumps"; i.e. two dimensional solitons/coherent structures which decay in all directions. In recent work we have found a new class of coherent structures which have more complicated interaction properties than the previously known lump solutions. Since the method of solution involves inverse scattering, we have been able to relate these new solutions to novel discrete eigenstates/eigenvalues-spectral singularities of the associated linear scattering problem, which in this case is the time dependent Schrödinger equation. Spectrally speaking these new solutions correspond to multiple poles associated with certain eigenfunctions of the nonstationary Schrödinger problem. The usual spectral description of, say, a two lump solution has two pairs of poles of the eigenfunction symmetrically located in the upper/lower half planes. We have shown, both by taking coalescing limits of the known lump solutions and by direct analysis of the scattering problem, that there are novel spectral configurations which have multiple poles in one of the half planes and simple poles in the other. This analysis has yielded new solutions to both the scattering problem, in this case the time dependent multidimensional Schrödinger problem, as well as for the KP equation when the correct parameterization for time is included. The solutions of the inverse scattering problem represent new "bound" state eigenfunctions and corresponding "reflectionless" potentials of the time dependent Schrödinger problem. The solutions of the KP equation are the time evolution of the potentials associated with the scattering problem. They correspond to a pair of lump type solutions which interact in an unusual way, not in the standard manner of

two lumps which interact without phase shift.

This work is important for anyone studying multidimensional nonlinear wave equations possessing coherent solutions. The underlying wave equations arise frequently in application as do the direct and inverse scattering problems. Both issues, solutions of mutlidimensional nonlinear wave equations and inverse scattering, have defense and civilian applications.

From another point of view, we have obtained a number of general results about the KP equations and related higher nonlinear modified KP equations—called the generalized KP (GKP) equation.

i) It turns out that the GKP equation generically has a discontinuous time derivative at the initial instant. In a recent paper (see paper #14 in 'publications accepted in journals' section of this report) we discuss this point and demonstrate how the temporal discontinuity is "resolved" by embedding the KP equation into a Boussinesq equation. This is analogous to way shock waves are "smoothed" by embedding an inviscid Burgers equation in a viscous Burgers equation.

ii) It turns out that in the modified KP—or more generally the GKP equation with cubic or higher order nonlinearities, the solitary wave solutions are unstable. In all cases we know of, such an instability results in wave collapse in finite time. We have developed a perturbation method which suggests that finite time blow up occurs. Our numerical simulations confirm this scenario.

Our studies of multidimensional integrable systems continue, with special attention directed at 2+1 discrete systems such as the 2+1 Toda system which is a partially discrete system (2 continuous, one discrete) variable. For the well known 2+1 Toda and Volterra equations we have recently obtained multidimensional lump type solitons, which decay in all directions. We are interested in obtaining fully discrete nonlinear wave systems which can be used for computational purposes as well as providing models of multidimensional lattice dynamics.

iii) Computational and Effective Chaos

We have been investigating the computational simulations of certain nonlinear equations analyzable by the inverse scattering transform (IST). We use these equations as prototypes since they are physically interesting systems about whose solutions and properties we have concrete analytical understanding. On the other hand, for the periodic boundary value problems we are considering, the analytical solutions are extremely complicated. Computationally speaking, these equations provide a vehicle by which: i) computational schemes can

be compared and ii) errors in the schemes can be detected. We have demonstrated that in certain circumstances computational chaos results. Since we are interested in the long time numerical integration of nonlinear systems, there is no existing theory of error analysis which describes the phenomena. We have been aided by the fact that we have obtained exact IST based discrete analogues of many of the continuous systems which, in practice, have proven to be excellent computational schemes.

The paradigm is the focusing NLS equation with periodic boundary conditions:

$$iu_t + u_{xx} + 2|u|^2 u = 0,$$

$$u(x,t) = u(x+L,t).$$

As mentioned above, we have a corresponding integrable discrete scheme,

$$iu_{nt} + \frac{u_{n+1} + u_{n-1} - 2u_n}{h^2} + |u_n|^2(u_{n+1} + u_{n-1}) = 0,$$

where h is the mesh size. For focusing NLS, there is a class of initial conditions which are linearly unstable. By changing a parameter in the initial condition, we can excite a number of linearly unstable modes; we call the number of unstable modes M. In our earlier work we have shown that a) for small values of M, standard numerical schemes (non-IST based schemes) break down and computational chaos results from truncation errors; b) for large values of M, the numerical chaos can even be induced by roundoff errors; c) for even initial data the chaos is explained by the demonstration of continual but temporally irregular crossings of unperturbed homoclinic manifolds (i.e. crossing of the NLS homoclinic manifolds). These manifolds are complicated. They have 2^n "sides" and allow for an extremely rich dynamical evolution; d) in our recent work, we have studied the case when the initial data is not even. In this latter situation, which is the generic case, the phase space is no longer foliated and we find that the solution to the perturbed NLS system can evolve from one "side" of the homoclinic manifold to another without crossing an unperturbed homoclinic manifold. The chaos we have observed in the latter case is depicted by irregular and continual changes of the velocity of the underlying periodic waves. The case of even initial data is typified by the periodic waves being essentially standing waves (no left/right velocity).

We believe that the computational chaos we have observed is, in fact, a manifestation of an important physical effect which should be observable in laboratory experiments. The NLS equation is well-known to govern the modulation of water waves in moderate-deep water, and modulational instability in nonlinear optics. When the waves are excited in a periodic manner, with small modulation, then the NLS equation with periodic initial data described above is the relevant equation. The Benjamin-Feir/modulational instability only

says that there are M unstable modes in the linearized version of the NLS equation. The NLS equation governs the long time evolution of the instability process. As mentioned above, our results regarding NLS, based on the extensive numerical and associated analysis, show that there is a significant difference in the long time dynamics depending on whether one excites a small number of unstable modes M (e.g M=1,2) or a large number (e.g M=5,6). In the former case, the dynamical evolution is explainable (and repeatable) in the context of NLS theory. In the latter situation, where round off error induces numerical chaos, the NLS equation is itself highly unstable in much the same way as coupled pendula nearly in the "up-position" are highly unstable. In the latter case, we believe that small physical perturbations should be capable of causing dynamical chaos in the evolution. In effect we are experimentally and dynamically close to the homoclinic manifold which induces the chaotic dynamics. We have discussed our results in detail with J. Hammack who is a well-known water wave experimentalist. He is planning on carrying out the experiments. We believe that these experiments will lead to interesting and important conclusions about the long time behavior of the Benjamin-Feir/modulational instabilities when the underlying waves are generated in a modulated periodic manner.

iv) Nonlinear Waves in Ferromagnetic Films

We have been studying a class of nonlinear waves in ferromagnetic media. Our motivation for these studies comes from the extensive experiments by Professor Carl Patton and his group in the Physics Department at Colorado State University. Patton's group has been investigating the generation and evolution of wave pulses which behave like solitons in thin film ferromagnets. In these experiments, an yttrium iron garnet (YIG) film is magnetized to saturation causing the dipoles of the ferromagnet to align. An external microwave signal is applied to the film. If the power is large enough, solitons are observed to form and propagate through the film.

We are interested in developing an effective theory governing the propagation of waves in such ferromagnetic media. There is an analogy with nonlinear optics. In optics the nonlinearity arises from the fact that the polarization is a nonlinear function of the electromagnetic field. In these magnetic systems the role of the polarization is played by the magnetization and the electric field is replaced by the magnetic field. The nonlinearity of the magnetic system is governed to leading order by a torque equation which describes the precession of dipoles in the magnetic media. The difference between optics and magnetics is important since the way the nonlinearity arises has a major effect on the amplitude equations (i.e. NLS type equations). The fact that we are modeling films means that we must consider three regions: two outside the film and the film itself. Outside the film we take the magnetostatic

approximation of Maxwell's equations; inside the film is where the nonlinearity (the torque equation) is applied in addition to the magnetostatic approximation of Maxwell's equations. The calculations turn out to be rather lengthy, but tractable.

Since Patton's experiments currently have a moderate transverse scale, our first efforts involve obtaining one dimensional amplitude equations. Let the film be aligned perpendicular to, say, the z-direction, and the propagation of the magnetic spin waves be along the longest direction, which we call the x-direction. The direction transverse to x, z is the y-direction. The case we have considered first is when the applied saturating field is in the z-direction. This is referred to as forward volume waves. There are two other cases of experimental interest; namely the so called backward volume case where the saturating field is applied in the x-direction, and the surface wave case where the saturating field is applied in the y-direction. With a student S. Mock, supported by an AFOSR AASERT grant, we have derived an amplitude equation governing modulated weakly nonlinear wavepackets for the forward volume case; the governing equation is the NLS equation. Earlier workers also derived an NLS equation for this situation. However, their work was based entirely on presuming a suitable form for the nonlinear dispersion relation; they did not derive the equation from first principles. Our calculations show that the coefficients in our NLS equation are significantly different from those derived earlier. We are comparing the experimental situation to our NLS system in order to deduce whether the difference in coefficients imply an improved fit to the laboratory data. Potential device applications envisaged are radar applications (the frequency range in which these signals operate is in the microwave regime) and the development of special purpose chips. Both of these applications have important defense and civilian implications.

We also have studied a different situation that can arise in the associated ferromagnetic systems; namely a case of a bulk system (thick ferromagnetic material) where the ferromagnetic region is assumed to dominate the entire space. It turns out that there is a relevant long wave regime in which the modified KP equation (ie GKP where the nonlinear is cubic) results (see also the discussion above).We believe that in this parameter regime localized waves will be strongly unstable and focussing will occur.

PERSONNEL SUPPORTED

- Faculty:

  Mark J. Ablowitz

- Post-Doctoral Associates:

  C. Schober, S. Chakaravarty

- Graduate Students:

  G. Biondini, S. Mock (via AFOSR ASSERT)

- Other (please list role)

  None

PUBLICATIONS

- **ACCEPTED**

  - **Books/Book Chapters**

    1. Numerical Stochasticity, Hamiltonian Integrators and the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, in *Three Dimensional Dynamical Systems,* Ed. Dr. Kandrup, Annals New York Academy of Sciences, (1994) 162-181.
    2. Chaos in Numerics, B.M. Herbst, G.J. Le Roux and M.J. Ablowitz, in *Numerical Analysis,* Eds. D.F. Griffiths and G.A. Watson, World Scientific, Singapore, (1996) 133-154.
    3. Discretizations, Integrable Systems and Computation, APPM† #306 (November 1996), (to be published in *Recent Developments in Soliton Theory–II*).

  - **Journals**

    1. Wave Collapse and Instability of Solitary Waves of a Generalized Nonlinear Kadomtsev-Petviashvili Equation, X.P. Wang, M.J. Ablowitz, and H. Segur, *Physica D,* **78** (1994) 241-265.
    2. Homoclinic Manifolds and Numerical Chaos in the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, *Mathematics and Computers in Simulation,* **37** (1994) 249-264.
    3. Effective Chaos in the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, *Contemporary Mathematics,* **172** (1994) 253-268.
    4. Multisoliton Interactions and Wavelength-Division-Multiplexing, S. Chakravarty, M.J. Ablowitz, J.R. Sauer, R.B. Jenkins, *Opt. Lett.,* **20** (1995) 136-138.
    5. Integrability, Computation and Applications, M.J. Ablowitz, S. Chakravarty and B.M. Herbst, *Acta Applicande Mathematicae,* **39** (1995) 5-37.

6. Data-Dependent Timing Jitter in WDMS Soliton Systems, R.B. Jenkins, J.R. Sauer, S. Chakravarty and M.J. Ablowitz, *Opt. Lett.,* **20** (1995) 1964-1966.
7. Numerical Simulation of Quasi-Periodic Solutions of the Sine-Gordon Equation, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *Physica D,* **87** (1995) 37-47.
8. Computational Chaos in the Nonlinear Schrödinger Equation Without Homoclinic Crossings, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *Physica A,* **228** (1996) 212-235.
9. On the Numerical Solution of the Sine-Gordon Equation I. Integrable Discretizations and Homoclinic Manifolds, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *J. Comp. Phys.,* **126** (1996) 299-314.
10. The Burgers Equation Under Deterministic and Stochastic Forcing, M.J. Ablowitz and S. De Lillo, *Physica D,* **92** (1996) 245-259.
11. On a 2+1 Volterra System, J. Villarroel, S. Chakravarty and M.J. Ablowitz, *Nonlinearity,* **9** (1996) 1113-1128.
12. Four-wave Mixing in Wavelength-division Multiplexed Soliton Systems—Damping and Amplification, M.J. Ablowitz, G. Biondini, S. Chakravarty, R.B. Jenkins and J.R. Sauer, *Optics Letters,* **21** (1996) 1646-1648.
13. The Nonlinear Schrödinger Equation: Asymmetric Perturbations, Traveling Waves and Chaotic Structures, M.J. Ablowitz, B.M. Herbst and C.M. Schober APPM† #254 (August 1995) (to be published *Mathematics and Computers in Simulation*).
14. Initial Time Layers and Kadomtsev-Petviashvili Type Equations, M.J. Ablowitz and X-P. Wang, APPM† #263 (December 1995) (to be published, *Stud. Appl. Math.*).
15. On the Numerical Solution of the Sine-Gordon Equation II. Performance of Numerical Schemes, M.J. Ablowitz, B.M. Herbst and C.M. Schober, APPM† #264 (November 1995) (to be published, *J. Comp. Phys.*).
16. Solutions to the Time Dependent Schrödinger and the Kadomtsev-Petviashvili Equations, M.J. Ablowitz and J. Villarroel, APPM† #277 (May 1996), (to be published, *Phys. Rev. Lett.*)
17. Four-wave Mixing in Wavelength-division Multiplexed Soliton Systems—Ideal Case, M.J. Ablowitz, G. Biondini, S. Chakravarty, R.B. Jenkins and J.R. Sauer, APPM† #283 (May 1996) (to be published, *J. Opt. Soc. B*).

– **Conferences**

1. Remarks on the Inverse Scattering Transform Associated with Toda Equations, M.J. Ablowitz and J. Villarroel, in *Quantum Inversion Theory and Applications*, Proceedings, Ed. H.V. von Geramb, *Lecture Notes in Physics*, Springer-Verlag, New York **427** (1994) 30-36.
2. Hamiltonian Integrators for the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, in *Proceedings of the 2nd IMACS Conference on Computational Physics,* Ed. J. Potvin, World Scientific, Singapore, (1994) 219-224.
3. On a New Class of Lump Type Solutions to the Kadomtsev-Petviashvili and Nonstationary Schrödinger Equations, M.J. Ablowitz, APPM† #305 (November 1996) (to be published in *Proceedings, Advances in soliton theory and its applications—The 30th anniversary of the Toda lattice, Yokohama, Japan*).

APPM†: Department of Applied Mathematics report

INTERACTIONS/TRANSITIONS

- Participation/Presentations At Meetings, Conferences, Seminars, Etc

1. Conference on Modern Group Analysis—Theory and Applications, Univ. of Witwatersrand, Johannesburg and University of Orange Free State, Blomenfontein, South Africa, "Are All Solitons Reductions of Self-Dual Yang Mills?", Jan.10-20, 1994.
2. Nonlinear Optics and Communications Workshop, Breckenridge, Colorado, "Multisoliton Interactions in Nonlinear Optical Fibers", April 11-12, 1994.
3. Symmetrics and Integrability of Difference Equations, Montreal, Canada, "Computational
Chaos in Integrable Systems–Truncation and Roundoff", May 23-24, 1994.
4. AFOSR Meeting on Computational and Physical Mathematics, Kirtland AFB, New Mexico, "Numerical Chaos in Coherent Systems", June 1-3, 1994.
5. Workshop on Twenty Years of the Nonlinear Schrödinger Equation and Recent Developments, Moscow, Russia, "Computational Chaos in the Nonlinear Schrödinger Equation", July 23-31, 1994.
6. University of Alberta, Edmonton, Alberta, "Computational Chaos in Integrable Systems", Sept. 8-10, 1994.

7. NEEDS Workshop, Los Alamos National Laboratory, Los Alamos, NM, "Integrability, Computation and Nonlinear Optics", Sept. 11-14, 1994.
8. Nonlinear Optics Workshop, Dept. of Mathematics, University of Arizona, "Soliton Interaction in Nonlinear Optics", October 9-11, 1994.
9. University of Tokyo, Physics Department, "Reductions of the Self Dual Yang Mills Equations and Novel Integrable Systems", Nov. 8, 1994; "Numerical Chaos: Truncation Roundoff", Nov. 9, 1994.
10. University of Tokyo, Applied Mathematics Department, "Novel Integrable Systems", Nov. 9, 1994.
11. University of Science and Technology, Hong Kong, "Computational Chaos in Integrable Systems", Nov. 16, 1994.
12. Hong Kong Polytechnic, Hong Kong, "Computational Chaos in Integrable Systems", Nov. 17, 1994.
13. University of Colorado, Program in Applied Mathematics, "1895-1995: Integrability and Applications", Feb. 10, 1995.
14. Colorado State University, Dept. of Mathematics, "1895-1995: Integrability and Applications", March 30, 1995.
15. PhD Course, KdV'95, University of Amsterdam, the Netherlands, "Numerical Computation of Integrable Systems I and II", April 21, 1995.
16. International Symposium, KdV'95, Amsterdam, the Netherlands, "1895-1995: Integrability and Applications", April 25, 1995.
17. Mathematics Department, University of Leeds, United Kingdom, "1895-1995: Integrability and Applications", April 27, 1995.
18. Mathematics Department, University of Loughborough, United Kingdom, "1895-1995: Integrability and Applications", May 5, 1995.
19. Workshop on Nonlinear Phenomena, Solitons and Symmetries, Kent University, Canterbury, UK "Integrability, Painlevé Equations and Novel Systems", May 9, 1995.
20. Workshop on Nonlinear Optics, Mathematics Department, University of Arizona, "Nonlinear Schrödinger Equations and Wavelength Division Multiplexing", October 2, 1995.
21. Department of Mathematics, Kent University, Canterbury, England "Computational and Effective Chaos in Integrable Systems", November 17,1995.

22. Department of Mathematics, Pondicherry University, Pondicherry, India, 6 lectures at the "Winter School in Nonlinear Systems", Topic: "Nonlinear Waves and the Inverse Scattering Transform", January 8-12, 1996.

23. Conference on Nonlinear Dynamics, School of Mathematics, University of New South Wales, Australia, "Computational Chaos in Integrable Systems", March 27-28, 1996.

24. Department of Mathematics, University of Sydney, Australia, "100 years of Integrability", April 3, 1996.

25. Institute for Theoretical Physics, summer school session on "Painlevé One Century Later", Cargèse, Corsica, "Painlevé Equations, Darboux-Halphen Systems and the Inverse Transform Method", 4 1/2 hours, June 3–12, 1996.

26. Workshop on Symmetries and Integrability, Kent University, Canterbury, England, "Solutions to the Time Dependent Schrödinger and the Kadomtsev-Petviashvili Equations", July 1–5, 1996.

27. Workshop in Nonlinear Optics, Mathematics Department, University of Arizona, Tucson, Arizona, "Wavelength Division Multiplexed Solitons and Four Wave Mixing", October 10–12, 1996.

28. International Symposium on Advances in soliton theory and its applications–The 30th anniversary of the Toda lattice, Yokohama National University, Yokohama, Japan, "On a New Class of Lump Type Solutions to the Kadomtsev-Petviashvili and Nonstationary Schrödinger Equations", December 1–4, 1996.

- **Consultative And Advisory Functions To Other Laboratories And Agencies**

- **Transitions**

NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES

None

HONORS/AWARDS

1. Sloan Foundation Fellowship: 1975–1977

2. John Simon Guggenheim Fellowship: 1984–85

3. University of Colorado Council of Research and Creative Work Fellowship: 1994–95